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## POTENTIALS OF THE PROBLEM OF STEADY-STATE OSCILLATIONS

OF THE GENERALIZED ASYMMETRICAL THERMOELASTICITY OF A COSSERAT MEDIUM
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UDC 539.3

The potentials of a simple layer and a double layer are determined, along with the volume potential of the problem of steady-state oscillations of the generalized thermoelasticity of a Cosserat medium; these potentials lead to integral equations of the second kind for the problem.

In the investigation of laser-induced thermal strains of optical materials it is necessary to treat more complex models than the classical versions in some cases. First of all, the finiteness of the heat propagation velocity must be considered in the study of heat-release processes associated with the pulsed application of optical radiation, i.e., it is necessary to go from classical to generalized thermomechanics [1]. Second, when the application of such materials as polycrystalline aggregates or an optical ceramic is considered, it is required to include not only the regular microstresses, but also couple stresses [2], necessitating the introduction of the Cosserat continuum model. The generalized thermoelasticity equations for a Cosserat medium have been derived previously [3]. An important special case is the problem of steady-state harmonic oscillations of a homogeneous isotropic polar-symmetrical medium. The system of equations in the complex amplitudes of the kinematic variables for an oscillation with frequency $\sigma$ has the form

$$
\begin{gather*}
(\mu+\alpha) \nabla^{2} \mathbf{u}+(\mu+\lambda-\alpha) \nabla \nabla \cdot \mathbf{u}+2 \alpha \nabla \times \boldsymbol{\omega}-v \Theta_{0} \nabla \vartheta+\rho \sigma^{2} \mathbf{u}+\mathbf{X}=0 \\
(\gamma+\varepsilon) \nabla^{2} \boldsymbol{\omega}+(\gamma+\beta-\varepsilon) \nabla \nabla \cdot \boldsymbol{\omega}+\left(/ \sigma^{2}-4 \alpha\right) \boldsymbol{\omega}+2 \alpha \nabla \times \mathbf{u}+\mathbf{Y}=0 \\
\frac{\Theta_{0} k}{1+i \sigma \tau_{0}} \nabla^{2} \vartheta-i \sigma m \Theta_{0}^{2} \vartheta-i \sigma \Theta_{0} v \nabla \cdot \mathbf{u}+w=0 \tag{1}
\end{gather*}
$$

One of the methods of analyzing and solving the boundary-value problems of thermoelasticity, particularly for regions bounded by noncanonical surfaces, is to reduce them to integral equations [4-6], specifically by means of potentials. It is first of all necessary in this connection to formulate the fundamental solutions [6] of the system (1).

We consider the problem of the action of a point force vector with amplitude value $a_{1}$ applied at the origin for $Y=0, w=0$. Invoking the regular solution of the homogeneous system (1) and the Fourier integral transform for the formulation of a particular solution of the inhomogeneous system, we obtain a solution of the system (1) subject to the Sommerfeld radiation condition:

$$
\begin{equation*}
u=U^{(1)} \cdot a_{1}, \quad \omega=\Omega^{(1)} \cdot a_{1}, \quad \vartheta=\Theta^{(1)} \cdot a_{1} \tag{2}
\end{equation*}
$$

Here the tensors $U^{(1)}, \Omega^{(1)}$ and the vector $\theta^{(1)}$ have the form

[^0]\[

$$
\begin{gather*}
\mathbf{U}^{(1)}=\frac{\mathrm{E}}{4 \pi(\mu+\alpha)}\left(\nabla^{3} p_{1}+\frac{I \sigma^{2}-4 \alpha}{\gamma+\varepsilon} p_{1}\right)+\frac{1}{4 \pi} V \nabla\left[\frac{1}{2 \mu+\lambda} p_{2}+\right. \\
\left.+\frac{m \Theta_{0}\left(\tau_{0} \sigma^{2}-i \sigma\right)}{k(2 \mu+\lambda)} p_{3}-\frac{1}{\mu+\alpha} p_{1}-\frac{1 \sigma^{2}-4 \alpha}{\gamma+\varepsilon)(\mu+\alpha)} p_{4}\right],  \tag{3}\\
\Omega^{(1)}=-\frac{1}{2 \pi} \frac{\alpha}{(\gamma+\varepsilon)(\mu+\alpha)} \nabla \times \mathbf{E} p_{1}, \\
\Theta^{(1)}=-\left(\tau_{0} \sigma^{2}-i \sigma\right) \frac{v}{4 \pi k(2 \mu+\lambda)} \nabla p_{1},
\end{gather*}
$$
\]

where the following notation has been introduced:

$$
\begin{gather*}
p_{1}=\frac{1}{|\mathbf{r}|\left(\xi_{1}^{2}-\xi_{2}^{2}\right)}\left\{\exp \left(i \xi_{2}|\mathbf{r}|\right)-\exp \left(i \xi_{1}|\mathbf{r}|\right)\right\},  \tag{4}\\
p_{2}=\frac{1}{|\mathbf{r}|\left(\eta_{1}^{2}-\eta_{2}^{2}\right)}\left\{\exp \left(i \eta_{2}|\mathbf{r}|\right)-\exp \left(i \eta_{1}|\mathbf{r}|\right)\right\}, \\
p_{3}=\frac{1}{|\mathbf{r}| \eta_{1}^{2} \eta_{2}^{2}}\left\{1+\frac{\eta_{2}^{2} \exp \left(i \eta_{1}|\mathbf{r}|\right)}{\eta_{1}^{2}-\eta_{2}^{2}}+\frac{\eta_{1}^{2} \exp \left(i \eta_{2}|\mathbf{r}|\right)}{\eta_{2}^{2}-\eta_{1}^{2}}\right\}, \\
p_{4}=\frac{1}{|\mathbf{r}| \xi_{1}^{2} \xi_{2}^{2}}\left\{1+\frac{\xi_{2}^{2} \exp \left(i \xi_{1}|\mathbf{r}|\right)}{\xi_{1}^{2}-\xi_{2}^{2}}+\frac{\xi_{1}^{2} \exp \left(i \xi_{2}|\mathbf{r}|\right)}{\xi_{2}^{2}-\xi_{1}^{2}}\right\},
\end{gather*}
$$

Here $\xi_{k}$ and $\eta_{k}$ represent the roots of the biquadratic equations

$$
\begin{gather*}
\xi^{4}+\xi^{2}\left[\frac{4 \mu \alpha}{(\gamma+\varepsilon)(\mu+\alpha)}-\frac{I \sigma^{2}}{\gamma+\varepsilon}-\frac{\rho \sigma^{2}}{\mu+\alpha}\right]+\frac{\rho \sigma^{2}\left(I \sigma^{2}-4 \alpha\right)}{(\gamma+\varepsilon)(\mu+\alpha)}=0, \\
\eta^{4}+\eta^{2}\left[\frac{i \sigma m \Theta_{0}\left(1+i \sigma \tau_{0}\right)}{k}+\frac{i v^{2} \Theta_{0} \sigma\left(1+i \sigma \tau_{0}\right)}{k(2 \mu+\lambda)}-\frac{\rho \sigma^{2}}{2 \mu+\lambda}\right]-\frac{i \rho \sigma^{3} m \Theta_{0}\left(1+i \sigma \tau_{0}\right)}{k(2 \mu+\lambda)}=0 . \tag{5}
\end{gather*}
$$

In the case of a point couple with amplitude value $a_{2}$ applied at the origin for $X=0$, $w=0$, the solution of the system (1) subject to the radiation condition can be written

$$
\begin{equation*}
\mathbf{u}=\mathbf{U}^{(2)} \cdot \mathbf{a}_{2}, \quad \boldsymbol{\omega}=\boldsymbol{\Omega}^{(2)} \cdot \mathbf{a}_{2}, \quad \boldsymbol{\vartheta}=0, \tag{6}
\end{equation*}
$$

where the tensors $U^{(2)}, \Omega^{(2)}$ have the form

$$
\begin{gather*}
\mathbf{U}^{(2)}=-\frac{\alpha}{2 \pi(\gamma+\varepsilon)(\mu+\alpha)} \nabla \times \mathbf{E} p_{1}, \\
\mathbf{\Omega}^{(2)}=\frac{1}{4 \pi(\gamma+\varepsilon)(2 \gamma+\beta)}\left\{(\varepsilon-\gamma-\beta) \nabla \nabla \nabla^{2} p_{5}+\right.  \tag{7}\\
\left.+\frac{4 \alpha^{2}-(\gamma+\beta-\varepsilon) \rho \sigma^{2}}{\mu+\alpha} \nabla \nabla p_{5}\right\}+\frac{\mathbf{E}}{4 \pi(\gamma+\varepsilon)}\left(\nabla^{2} p_{1}+\frac{\rho \sigma^{2}}{\mu+\alpha} p_{1}\right) .
\end{gather*}
$$

Here, in addition to (4) and (5), we have introduced the notation

$$
\begin{gather*}
p_{5}=\frac{1}{|r|}\left\{\frac{\exp \left(i \xi_{1}|\mathbf{r}|\right)}{\left(\xi_{1}^{2}-\xi_{2}^{2}\right)\left(\xi_{1}^{2}-\xi_{3}^{2}\right)}+\frac{\exp \left(i \xi_{2} \mid \mathbf{r}\right)}{\left(\xi_{2}^{2}-\xi_{1}^{2}\right)\left(\xi_{2}^{2}-\xi_{3}^{2}\right)}+\frac{\exp \left(i \xi_{3}|\mathbf{r}|\right)}{\left(\xi_{3}^{2}-\xi_{1}^{2}\right)\left(\xi_{3}^{2}-\xi_{2}^{2}\right)}\right\}, \\
\xi_{3}^{2}=\frac{I \sigma^{2}-4 \alpha}{2 \gamma+\beta} . \tag{8}
\end{gather*}
$$

Finally, under the action of a point heat-release source at the origin with amplitude value $a_{3}$ the solution of equations (1) subject to the radiation condition has the form

$$
\begin{equation*}
\mathbf{u}=\mathbf{U}^{(3)} a_{3}, \quad \omega=0, \quad \vartheta=a_{3} \Theta^{(3)} \tag{9}
\end{equation*}
$$

where

$$
\begin{gather*}
\mathrm{U}^{(3)}-\frac{v\left(1+i \sigma \tau_{0}\right)}{4 \pi k(2 \mu+\lambda)} \nabla p_{2} ; \\
\Theta^{(3)}=\frac{1+i \sigma \tau_{0}}{4 \pi k \Theta_{0}}\left(\nabla^{2} p_{2}+\frac{\rho \sigma^{2}}{2 \mu+\lambda} p_{2}\right) . \tag{10}
\end{gather*}
$$

Consequently, if a point force, a point couple, and a point heat-release source act at the origin, the system (1) has the following solution regular everywhere except at the origin:

$$
\begin{gather*}
\mathbf{u}^{(1)}=U^{(1)} \cdot \mathrm{a}_{1}+\mathbf{U}^{(2)} \cdot \mathrm{a}_{2}+a_{3} \mathrm{U}^{(3)} \\
\boldsymbol{\omega}^{(1)}=\boldsymbol{\Omega}^{(1)} \cdot \mathbf{a}_{\mathbf{1}}+\mathbf{\Omega}^{(2)} \cdot \mathbf{a}_{2}  \tag{11}\\
\boldsymbol{\vartheta}^{(1)}=\boldsymbol{\Theta}^{(1)} \cdot \mathbf{a}_{\mathbf{1}}+a_{3} \Theta^{(3)}
\end{gather*}
$$

We denote the singular part of a tensor or vector by the subscript $s$. It is readily verified that the singular part of the solution (11) does not depend on $\sigma$ :

$$
\begin{gather*}
\mathbf{U}_{s}^{(1)}=\frac{3 \mu+\lambda+\alpha}{8 \pi|\boldsymbol{r}|(2 \mu+\lambda)(\mu+\alpha)} E+\frac{\mu+\lambda-\alpha}{8 \pi|\mathbf{r}|^{3}(2 \mu+\lambda)(\mu+\alpha)} \mathbf{r r} \\
\mathbf{\Omega}_{s}^{(2)}=\frac{3 \gamma+\beta+\varepsilon}{8 \pi|\mathbf{r}|(2 \gamma+\beta)(\gamma+\varepsilon)} \mathbf{E}+\frac{\gamma+\beta-\varepsilon}{8 \pi|\boldsymbol{r}|^{3}(2 \gamma+\beta)(\gamma+\varepsilon)} \mathbf{r r} \\
\Theta_{s}^{(3)}=\frac{1}{4 \pi k \Theta_{0}|\mathbf{r}|},  \tag{12}\\
\mathbf{U}_{s}^{(2)}=0, \quad \mathbf{\Omega}_{s}^{(1)}=0, \quad \mathbf{U}_{s}^{(3)}=0, \quad \Theta_{s}^{(1)}=0
\end{gather*}
$$

Apart from notational differences, equations (12) coincide with the expressions given in [6] for the singular parts of the fundamental solutions for the couple elasticity.

We now introduce the tensors $\Phi^{(1)}, \Phi^{(2)}, \Psi^{(1)}, \Psi^{(2)}$, the vectors $\Phi(3), p^{(1)}$, and the scalar $P^{(3)}$, which are related to $U(1), U(2), U(3), \Omega^{(1)}, \Omega^{(2)}, \theta^{(1)}, \theta^{(3)}$ by the expressions

$$
\begin{align*}
& \boldsymbol{\Phi}^{(1)}=(\mu+\alpha) \mathbf{n} \cdot \nabla \mathbf{U}^{(1)}+(\mu-\alpha) \nabla \mathbf{n} \cdot \mathbf{U}^{(1)}+\lambda \mathbf{n} \nabla \cdot \mathbf{U}^{(1)}+\frac{\sigma^{2} v}{k}\left(1+i \sigma \tau_{0}\right) n \mathbf{U}^{(3)} . \\
& \boldsymbol{\Phi}^{(2)}=(\gamma+\varepsilon) \mathbf{n} \cdot \nabla \mathbf{U}^{(2)}+\cdots(\gamma-\varepsilon) \nabla \mathbf{n} \cdot \mathbf{U}^{(2)}+\beta \mathbf{n} \nabla \cdot \mathbf{U}^{(2)}+2 \alpha \mathbf{n} \times \mathbf{U}^{(2)},  \tag{13}\\
& \boldsymbol{\Phi}^{(3)}=\mathbf{n} \cdot \nabla \mathbf{U}^{(3)}, \quad \mathbf{\Psi}^{(1)}=(\mu+\alpha) \mathbf{n} \cdot \nabla \mathbf{\Omega}^{(1)}+(\mu-\alpha) \nabla \mathbf{n} \cdot \mathbf{\Omega}^{(1)}+\lambda \mathbf{n} \nabla \cdot \mathbf{\Omega}^{(1)}, \\
& \Psi^{(2)}=(\gamma+\varepsilon) \mathbf{n} \cdot \nabla \mathbf{\Omega}^{(2)}+(\gamma-\varepsilon) \nabla \mathbf{n} \cdot \mathbf{\Omega}^{(2)}+\beta n \nabla \cdot \mathbf{\Omega}^{(2)}+2 \alpha \mathbf{n} \times \mathbf{\Omega}^{(2)}, \\
& p^{(1)}=(\mu+\alpha) \mathbf{n} \cdot \nabla^{(1)}+(\mu-\alpha) \nabla \mathbf{n} \cdot \Theta^{(1)}+\lambda \mathbf{n} \nabla \cdot \Theta^{(1)} \\
& +\frac{\sigma^{2} v}{k}\left(1+i \sigma \tau_{0}\right) \mathbf{n} \Theta^{(3)}, \quad P^{(3)}=\mathbf{n} \cdot \nabla^{(3)} .
\end{align*}
$$

It can be directly verified that

$$
\begin{gather*}
\mathbf{u}^{(2)}=\mathbf{b}_{1} \cdot \mathbf{\Phi}^{(1)}+\mathbf{b}_{2} \cdot \boldsymbol{\Phi}^{(2)}+b_{3} \mathbf{\Phi}^{(3)}: \boldsymbol{\omega}^{(2)}=\mathbf{b}_{1} \cdot \mathbf{\Psi}^{(1)}+\mathbf{b}_{2} \cdot \Psi^{(2)} \\
\mathbf{母}^{(2)}=\mathbf{b}_{1} \cdot \mathbf{P}^{(1)}+b_{3} p^{(3)} \tag{14}
\end{gather*}
$$

represents the solution of the homogeneous system (1) everywhere except at the origin for any constants $b_{1}, b_{2}$, and $b_{3}$. The solutions (11) and (14) can be used to find expressions for the potentials. If we consider the domain $V$ bounded by a closed Lyapunov surface $S$, we can introduce the volume potential

$$
\begin{gather*}
\mathbf{u}^{\mathbf{I}}\left(\mathbf{r}_{0}\right)=\int_{V}\left\{\mathbf{U}^{(\mathbf{(})}(\mathbf{R}) \cdot \mathbf{a}_{1}(\mathbf{r})+\mathbf{U}^{(2)}(\mathbf{R}) \cdot \mathbf{a}_{2}(\mathbf{r})+a_{3}(\mathbf{r}) \mathbf{U}^{(3)}(\mathbf{R})\right\} d V(\mathbf{r}) \\
\boldsymbol{\omega}^{1}\left(\mathbf{r}_{0}\right)=\int_{V}\left\{\mathbf{\Omega}^{(1)}(\mathbf{R}) \cdot \mathbf{a}_{1}(\mathbf{r})+\mathbf{\Omega}^{(2)}(\mathbf{R}) \cdot \mathbf{a}_{2}(\mathbf{r})\right\} d V(\mathbf{r}) \tag{15}
\end{gather*}
$$

$$
\begin{equation*}
\vartheta^{\mathrm{I}}\left(\mathrm{r}_{0}\right)=\int_{V}\left\{\Theta^{(1)}(\mathbf{R}) \cdot \mathrm{a}_{1}(\mathrm{r})+a_{3}(\mathbf{r}) \Theta^{(3)}(\mathbf{R})\right\} d V(\mathbf{r}), \tag{15}
\end{equation*}
$$

where the following notation is introduced:

$$
\begin{equation*}
\mathbf{R}^{2}=\left(\mathbf{r}-\mathbf{r}_{\mathbf{0}}\right)^{\mathbf{2}}, \quad \mathbf{r}=x_{k} \mathbf{e}_{k}, \quad \mathbf{r}_{0}=x_{0 k} \mathbf{e}_{k}, \tag{16}
\end{equation*}
$$

and the differential operations in the integrands are naturally taken with respect to the variables $x k$. Moreover, we introduce the potentials of a simple layer

$$
\begin{gather*}
\mathbf{u}^{11}\left(\mathbf{r}_{0}\right)=\int_{S}\left\{U^{(1)}(\mathbf{R}) \cdot \mathbf{a}_{1}(\mathbf{r})+U^{(2)}(\mathbf{R}) \cdot \mathbf{a}_{2}(\mathbf{r})+a_{3}(\mathbf{r}) U^{(3)}(\mathbf{R})\right\} d S(\mathbf{r}), \\
\boldsymbol{\omega}^{11}\left(\mathbf{r}_{0}\right)=\int_{S}\left\{\mathbf{\Omega}^{(1)}(\mathbf{R}) \cdot \mathbf{a}_{1}(\mathbf{r})+\mathbf{\Omega}^{(2)}(\mathbf{R}) \cdot \mathbf{a}_{2}(\mathbf{r})\right\} d S(\mathbf{r})  \tag{17}\\
\vartheta^{11}\left(\mathbf{r}_{0}\right)=\int_{S}\left\{\Theta^{(1)}(\mathbf{R}) \cdot \mathbf{a}_{1}(\mathbf{r})+a_{3}(\mathbf{r}) \Theta^{(3)}(\mathbf{R})\right\} d S(\mathbf{r})
\end{gather*}
$$

and a double layer

$$
\begin{align*}
& \mathbf{u}^{I I I}\left(\mathbf{r}_{0}\right)=\int_{S}\left\{\mathbf{b}_{1}(\mathbf{r}) \cdot \Phi^{(1)}(\mathbf{R})+\mathbf{b}_{2}(\mathbf{r}) \cdot \Phi^{(2)}(\mathbf{R})+b_{3}(\mathbf{r}) \Phi^{(3)}(\mathbf{R})\right\} d S(\mathbf{r}) \\
& \omega^{I I I}\left(\mathbf{r}_{0}\right)==\int_{S}\left\{\mathbf{b}_{1}(\mathbf{r}) \cdot \Psi^{(1)}(\mathbf{R})+\mathbf{b}_{2}(\mathbf{r}) \cdot \Psi^{(2)}(\mathbf{R})\right\} d S(\mathbf{r})  \tag{18}\\
& \vartheta^{I I I}\left(\mathbf{r}_{0}\right)=\int_{S}\left\{\mathbf{b}_{1}(\mathbf{r}) \cdot \mathbf{P}^{(1)}(\mathbf{R})+b_{\mathbf{3}}(\mathbf{r}) P^{(3)}(\mathbf{R})\right\} d S(\mathbf{r})
\end{align*}
$$

Inasmuch as the singular parts of the potentials (15), (17), and (18), apart from notational differences, coincide with the singular parts of previously investigated potentials [6], the potentials (15), (17), (18) and their normal derivatives on the surface have the same continuity properties or type of discontinuity as their classical analogs. This fact permits them to be used to derive integral equations of the second kind by exact analogy with the previously investigated potentials.

It is important to note that the classical thermoelastic potentials of a Cosserat medium can be obtained by setting $\tau_{0}=0$ everywhere, although, of course, it would be possible to consider the classical analog of the system (1) from the outset in order to obtain the indicated potentials, and then to go over to Eqs. (15), (17), and (18) by the correspondence principle [7].

As an example, we consider the first outer problem for the system of equations (1), i.e., we assume that the elastic medium occupies a volume $V$ bounded by a closed Lyapunov surface $S$, on which the following boundary conditions are specified:

$$
\begin{equation*}
\left.\mathbf{u}\right|_{S}=\boldsymbol{\varphi}\left(\mathbf{r}_{0}\right),\left.\quad \omega\right|_{S}=\boldsymbol{\psi}\left(\mathbf{r}_{0}\right),\left.\quad \boldsymbol{v}\right|_{S}=g\left(\mathbf{r}_{0}\right) . \tag{19}
\end{equation*}
$$

The solution of such a problem can be sought in the form of the sum of the potentials (15) and (18). Since the double-layer potential (18) behaves near the boundary in the same way as the corresponding static potential [6], the problem is reducible to the system of integral equations

$$
\begin{gather*}
-\mathbf{b}_{1}\left(\mathbf{r}_{0}\right)+\int_{S}\left\{\mathbf{b}_{1}(\mathbf{r}) \cdot \Phi^{(1)}(\mathbf{R})+\mathbf{b}_{\mathbf{2}}(\mathbf{r}) \cdot \boldsymbol{\Phi}^{(2)}(\mathbf{R})+b_{3}(\mathbf{r}) \boldsymbol{\Phi}^{(3)}(\mathbf{R})\right\} d S(\mathbf{r})= \\
=\boldsymbol{\varphi}\left(\mathbf{r}_{0}\right)-\int_{V}\left\{\mathbf{U}^{(1)}(\mathbf{R}) \cdot \mathbf{a}_{1}(\mathbf{r})+\mathbf{U}^{(2)}(\mathbf{R}) \cdot \mathbf{a}_{2}(\mathbf{r})+a_{3}(\mathbf{r}) \mathbf{U}^{(3)}(\mathbf{R})\right\} d V(\mathbf{r}) \\
-\mathbf{b}_{2}\left(\mathbf{r}_{0}\right)+\int_{S}\left\{\mathbf{b}_{1}(\mathbf{r}) \cdot \Psi^{(1)}(\mathbf{R})+\mathbf{b}_{2}(\mathbf{r}) \cdot \mathbf{\Psi}^{(2)}(\mathbf{R})\right\} d S(\mathbf{r})=  \tag{20}\\
=\boldsymbol{\Psi}\left(\mathbf{r}_{0}\right)-\int_{V}\left\{\mathbf{\Omega}^{(1)}(\mathbf{R}) \cdot \mathbf{a}_{1}(\mathbf{r})+\mathbf{\Omega}^{(2)}(\mathbf{R}) \cdot \mathbf{a}_{2}(\mathbf{r})\right\} d V(\mathbf{r}), \\
-\mathbf{b}_{3}\left(\mathbf{r}_{0}\right)+\int_{S}\left\{\mathbf{b}_{1}(\mathbf{r}) \cdot \mathbf{P}^{(1)}(\mathbf{R})+b_{3}(\mathbf{r}) P^{(3)}(\mathbf{R})\right\} d S(\mathbf{r})=g\left(\mathbf{r}_{0}\right)-\int_{V}\left\{\boldsymbol{\Theta}^{(1)}(\mathbf{R}) \cdot \mathbf{a}_{1}(\mathbf{r})+\mathbf{a}_{3}(\mathbf{r}) \Theta^{(3)}(\mathbf{R})\right\} d V(\mathbf{r})
\end{gather*}
$$

Singular integral equations can also be obtained for other boundary-value problems. All of these systems of integral equations are analogous to those discussed in [6] as part of a study of other branches of the theory of elasticity. Consequently, the validity of the Fredholm alternative hypothesis can be proved for them, and previously developed methods of solution [8, 9] can be used.

## NOTATION

$u$, complex amplitude of the displacement vector; $\omega$, complex amplitude of the microrotation vector; $\vartheta$, complex amplitude of the relative deviation of the absolute temperature $\theta$ from the initial value $\theta_{0} ; \mu, \lambda, \alpha, \gamma, \varepsilon, \beta, \nu, m, c o n s t a n t s$ characterizing the elastic and thermophysical properties of the material; $\sigma$, oscillation frequency; $\tau_{0}$, time constant characterizing the heat propagation velocity; $\rho$, density of the material; $I$, quantity characterizing the inertial properties of the material in microrotations; $k$, thermal conductivity; $X$, complex amplitude of the external body-force vector; $Y$, complex amplitude of the external body couples; w, complex amplitude of the heat-release density; $E$, unit tensor; r, radius vector of point with coordinates $x_{n} ; r_{0}$, radius vector of point with coordinates $x_{0 n}$; $e_{n}$, unit vectors of orthogonal coordinate system.

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[^0]:    Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 5, pp, 844-849, November, 1985. Original article submitted July $27,1984$.

